WJEC CBAC

# GCE AS/A level

0977/01

## MATHEMATICS – FP1
### Further Pure Mathematics

P.M. WEDNESDAY, 18 June 2014

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.


CJ*(S14-0977-01)

**1.** *(a)* Differentiate $\frac{1}{x^2}$ from first principles. [6]

*(b)* The function $f$ is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = (\sec x)^x.$$

Obtain an expression for $f'(x)$, simplifying your answer. [4]

**2.** *(a)* Find an expression in its simplest form for

$$\sum_{r=1}^{n} r(r+3).$$ [4]

*(b)* Given that the sum of the first $n$ terms of another series is $n(n + 3)$, obtain an expression for the $n$th term of the series. [3]

**3.** Consider the following equations.

$$\begin{aligned} x + 2y + 4z &= 3, \\ x - y + 2z &= 4, \\ 4x - y + 10z &= k. \end{aligned}$$

Given that the equations are consistent,

*(a)* find the value of $k$, [5]

*(b)* determine the general solution of the equations. [3]

**4.** The complex number $z$ is given by

$$z = \frac{1+2\mathrm{i}}{1-\mathrm{i}}.$$

Find the modulus and the argument of $z$. [6]

**5.** The roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0$$

are denoted by $\alpha, \beta, \gamma$.

*(a)* Find the cubic equation whose roots are $\beta\gamma, \gamma\alpha, \alpha\beta$. [6]

*(b)* Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

Deduce the number of real roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0,$$

justifying your answer. [4]

**6.** The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 2 & 3 \\ -1 & 1 & 1 \\ 2 & \lambda & 2 \end{bmatrix}.$$

*(a)* Find the values of $\lambda$ for which **A** is singular. [4]

*(b)* Given that $\lambda = -1$,

(i) find the adjugate matrix of **A**,
(ii) find the inverse of **A**. [5]

**7.** The transformation *T* in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point $(x, y)$ is transformed to the point $(x + 1, y + 2)$, followed by a reflection in the $y$-axis.

*(a)* Show that the matrix representing *T* is

$$\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

[5]

*(b)* Find the equation of the image under *T* of the line $y = 2x + 1$. [4]

**8.** Using mathematical induction, prove that

$$\sum_{r=1}^{n} \left(r \times 2^{r-1}\right) = 1 + 2^{n}(n-1),$$

for all positive integers $n$. [7]

**9.** The complex numbers $z$ and $w$ are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(z-1).$$

*(a)* Obtain expressions for $u$ and $v$ in terms of $x$ and $y$. [4]

*(b)* The point *P* moves along the line $x + y = 0$. Find the equation of the locus of *Q*. [5]

**END OF PAPER**